Pergamon

PII: S0021-8928(96)00097-4

J. Appl. Maths Mechs, Vol. 60, No. 5, pp. 769–775, 1996

0021-8928/96 $24.00+0.00

DYNAMICS OF A LIQUID WITH MOMENTUM ANISOTROPY†

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(*Received* 30 *January* 1995)

A dynamical model of a liquid with momentum anisotropy (LMA) is constructed using the ideas and principles of the momentum mechanics of continuous media. The equations for a lubricating layer are derived in the thin-layer approximation. Shear flow between parallel surfaces is considered. It is shown that a liquid with momentum anisotropy exhibits a dimensional effect in its viscosity and a kinematic ordering in the orientation of the long axes of the molecules. The approach which has been developed is promising for an adequate description of the theology of thin boundary layers which are formed close to a solid surface and determine the fundamental friction laws.

Experiments convincingly show that many liquids form boundary (or presurface) layers with a thickness of ~20–50 nm, in which the molecules are orientationally ordered around a solid surface. In the opinion of the Derjaguin school [1–4], certain liquids close to a solid surface form a new phase—an epitropic liquid crystal, and, moreover, it has been shown in [1–4] that the effect of a solid surface on the liquid properties extends up to distances of several microns.

The interest in this problem is due to the fact that the phenomena occurring in boundary layers determine the physical basis of many important technological processes such as flotation, coagulation, the stability of disperse and colloidal systems, friction, etc. The latter phenomenon is of interest in its own right and is deserving of special consideration. It is customary to associate the carrying capacity of friction points with the dynamics of a Newtonian fluid in a wedge-shaped gap (the oily wedge effect) in which a dynamic disjoining pressure arises. The adhesive forces between the liquid and the solid surface are substantial in narrow gaps of the order of a micron and less, in which the Navier–Stokes equations do not hold. However, the problem of taking account of their effect on the interaction of solid surfaces in a quantitative manner has not yet been solved.

For this reason, the fundamental issue on the nature of the lubricating action of oils and the role of liquid crystal-like structures in their microrheology again arises. It is known that smectic liquid crystals also occur in substances which have long been used as additives in lubricating oils and greases. These are surfactants: colloidal micelle-forming solutions of amphiphilic compounds which yield layer structures. The salts of fatty acids may serve as an example.

Hence the development of a hydrodynamic theory of boundary layers, which would enable us to describe their rheological behaviour and the kinematic orientation induced by the solid surface adequately, is urgent. Such a theory can be constructed using the ideas and principles of the momentum mechanics of a continuous medium [5–7] and, in particular, of a model of a liquid with momentum anisotropy.

1. THE EQUATIONS OF MOTION

The equations of motion of a liquid with momentum anisotropy (LMA) can be written in the form

$$\frac{d\rho}{dt} = -\rho v_{k,k}, \quad \rho\frac{dv_i}{dt} = \sigma_{ik,k} + \rho f_i$$
$$\rho\frac{dS_i}{dt} = \mu_{ik,k} - \sigma_{nm}\varepsilon_{inm} + \rho m_i \tag{1.1}$$

Here $S_i = J_{ik}\dot{\Omega}_k$, σ_{ik} and μ_{ik} are the asymmetric force and torque stress tensors, ρf_i and ρm_i are the densities of the bulk forces and moments, v_i, $\dot{\Omega}_i$ and J_{ik}, are the translational velocity, the intrinsic angular

†*Prikl. Mat. Mekh.* Vol. 60, No. 5, pp. 778–785, 1996.

velocity and the moment of inertia of an elementary part of the medium, and ε_{inm} is the Levi–Civita tensor.

In momentum, liquids as in liquid crystals, the molecules themselves, which rotate about their own centres of inertia, are the carriers of the intrinsic angular momentum S_i. Calculations show that the spin angular momentum only plays an important role at very high deformation rates (high acoustic frequencies)† and turns out to be negligibly small at the usual deformation rates. The intrinsic angular velocity $\dot{\Omega}_i$ in a LMA is made up of the velocity of rotation of the anisotropic direction $\dot{\Omega}_i^+$ (of the local axis of symmetry of the liquid at a given point L_i) and the velocity of rotation about the axis of anisotropy $\dot{\Omega}_i^-$

$$\dot{\Omega}_i = \dot{\Omega}_i^+ + \dot{\Omega}_i^-, \quad \dot{\Omega}_i^+ = L_n \frac{dL_m}{dt} \varepsilon_{inm}, \quad \dot{\Omega}_i^- = L_i \dot{\Psi}, \quad L_i L_i = 1 \tag{1.2}$$

The rotational degrees of freedom of a LMA are described by three quantities, that is, by two components of the vector L_i and the parameter $\dot{\Psi}$.

In momentum mechanics (the Cosserat model with free rotation), the intrinsic angular velocity $\dot{\Omega}_i$ is an independent quantity which does not depend on $\dot{\omega} \equiv \nabla \times \mathbf{v}/2$, which describes the rotation of a part of the medium as a whole.

The local deformation of a part of the medium in momentum hydrodynamics is characterized by two tensor quantities

$$\dot{e}_{ik} = \tfrac{1}{2}(v_{i,k} + v_{k,i}), \quad \dot{r}_{ik} = \dot{\Omega}_{i,k} \tag{1.3}$$

Material relations, that is, the laws associating the dynamic quantities σ_{ik}, μ_{ik} and the kinematic quantities $\dot{e}_{ik}$, $\dot{r}_{ik}$ and the $(\dot{\Omega}_i - \dot{\omega}_i)$ quantities for a LMA can be obtained in the same way as in the case of liquid crystals [8] using the first and second laws of thermodynamics, the principle of invariance to rigid rotation, the Onsager principle, the conditions of material symmetry as well as firmly established experimental data concerning the rheological properties of the boundary layers. In the case of low deformation rates (apart from linear terms in the expansion in terms of $\dot{e}_{ik}$, $\dot{r}_{ik}$), one can write

$$\begin{aligned} \sigma_{(ik)} &= -p\delta_{ik} + A_{(ik)(mn)}\dot{e}_{nm} + A_{(ik)j}(\dot{\Omega} - \dot{\omega})_j \\ \sigma_{[ik]} &= A_{[ik](mn)}\dot{e}_{nm} + A_{[ik]j}(\dot{\Omega} - \dot{\omega})_j \\ \mu_{ik} &= \Theta_{ikmn}\dot{r}_{mn} + L_i\mu_k \end{aligned} \tag{1.4}$$

Here, $A_{[ik]j} = A_{[ik]mn}\varepsilon_{nmj}$; $A_{(ik)j} = A_{(ik)mn}\varepsilon_{nmj}$, (ik) and $[ik]$ are the symbols of symmetrization and of antisymmetrization, and A_{iknm} and Θ_{ikmn} are the shear viscosity and torque viscosity tensors. The explicit form of these tensors can be found if the material symmetry of the anisotropic liquid is taken into account.

We assume that the LMA locally (at each point) possesses cylindrical symmetry (L_i is the unit vector along the axis of symmetry) and, in addition, that it has a plane of mirror symmetry perpendicular to the axis of cylindrical symmetry. In other words, we assume that L_i and $-L_i$ are physically indistinguishable. In this case, the tensors A_{iknm} and Θ_{ikmn} can be represented in terms of the dyads $L_i L_k$ and the absolute tensors δ_{ik} and ε_{mns}. In all, each tensor will contain eight independent parameters which can be interpreted as the coefficients of viscosity under certain flow conditions. Finally, the material relations for the LMA have the form

$$\begin{aligned} \sigma_{ik} &= -p\delta_{ik} + a_1\dot{e}_{ik} + \frac{1}{2}(a_2 + a_6)\dot{e}_{nk}L_nL_i + \frac{1}{2}(a_2 - a_6)\dot{e}_{in}L_nL_k + \\ &+ (a_3L_iL_k + a_4\delta_{ik})\dot{e}_{nm}L_nL_m + (a_5\delta_{ik} + a_4L_iL_k)\dot{e}_{nn} + \\ &+ a_7(\dot{\Omega} - \dot{\omega})_j\varepsilon_{jik} + \frac{1}{2}(a_6 + a_8)L_iN_k + \frac{1}{2}(a_6 - a_8)L_kN_i \\ \sigma_{[ik]} &= \frac{1}{2}a_6(\dot{e}_{kn}L_nL_i - \dot{e}_{in}L_nL_k) + a_7(\dot{\Omega} - \dot{\omega})_j\varepsilon_{jik} + \frac{1}{2}a_8(L_iN_k - L_kN_i) \\ \mu_{ik} &= \Theta_1\dot{r}_{ik} + \frac{1}{2}(\Theta_2 + \Theta_6)\dot{r}_{nk}L_nL_i + \frac{1}{2}(\Theta_2 - \Theta_6)\dot{r}_{in}L_nL_k + \end{aligned} \tag{1.5}$$

†AERO E. L., Theory of sound propagation in liquid crystals. Candidate dissertation, 15.74.03, Leningrad, 1974.

$$+(\Theta_3 L_i L_k + \Theta_4 \delta_{ik}) \dot{r}_{mn} L_m L_n + (\Theta_4 L_i L_k + \Theta_5 \delta_{ik}) \dot{r}_{nn} +$$
$$+\Theta_7 (\dot{r}_{ni} L_n L_k + \dot{r}_{kn} L_n L_i) + \Theta_8 \dot{r}_{ki} + L_i \mu_k$$
$$\mu_k = (a_9 \delta_{ki} + a_{10} L_k L_i n_i)$$
$$n_i = L_j \dot{\Omega}_{j,i} - \dot{\Omega}_j L_{j,i}, \quad N_i = \frac{dL_i}{dt} + L_n \dot{\omega}_n \varepsilon_{inm}$$

By virtue of the mirror symmetry, the tensors A_{ikmn} and Ω_{ikmn} only contain the even dyads ($L_i L_k$, $L_i L_k L_m L_n$) and do not contain the odd dyads ($L_i, L_i L_k, L_m$).

In the case of incompressible media, Eq. (1.1), together with the material relations (1.5), form a closed system of seven equations for the seven quantities (v_i, L_i, Ψ, p). In the case of compressible media, the density ρ is added to the required quantities and the equation of state of the medium is added to the conservation laws.

The system of equations obtained, as a special case, contains the equations of momentum hydrodynamics [6, 7]. They can be obtained if the tensors A_{ikmn} and Θ_{ikmn} are averaged over all orientations and account is taken of the fact that

$$\langle L_i L_k \rangle = \frac{1}{3}\delta_{ik}, \quad \langle L_i L_k l_m L_n \rangle = \frac{1}{15}(\delta_{ik}\delta_{nm} + \delta_{im}\delta_{kn} + \delta_{in}\delta_{km})$$

They also contain the equations of an anisotropic liquid (the Ericksen–Leslie model). Actually, if the spin moment S_i, the bulk moment ρm_i and the viscous torque stresses are neglected in the last equation of (1.1), that is, if it is assumed that

$$S_i = 0, \quad \rho m_i = 0, \quad \mu_{ik} = 0$$

then the force stress tensor becomes symmetric and the last two equations of (1.1) can be written in the form of the system

$$\rho \frac{dv_i}{dt} = \sigma_{(ik),k} + \rho f_i$$
$$\frac{dL_i}{dt} + L_n \dot{\omega}_m \varepsilon_{inm} = \lambda (L_i \dot{e}_{kn} L_k L_n - \dot{e}_{in} L_n), \quad \lambda = \frac{a_6}{2a_7 + a_8}$$

which was proposed for the first time by Ericksen in the case of a momentum-free anisotropic liquid.

2. THE LMA EQUATIONS FOR A LUBRICATING LAYER

Since, later, we primarily have hydrodynamic frictional problems in mind, it is worth obtaining the equations of motion for a lubricating layer. It is seen from (1.5) that taking account of the orientational ordering of the molecules of a liquid leads to non-linear material relations.

For this reason, the equations of motion turn out to be non-linear and very complex even in the case of creeping flows.

We introduce the dimensionless coordinates $\bar{x}$, $\bar{y}$, $\bar{z}$ and $\bar{t}$

$$x = l\bar{x}, \quad y = \delta\bar{y}, \quad z = l\bar{z}, \quad t = \frac{\delta}{U}\bar{t} \tag{2.1}$$

the dimensionless components of the velocity $\bar{u}$, $\bar{v}$, $\bar{w}$ and the dimensionless time $\bar{p}$

$$u = U_1\bar{u}, \quad v = V_l\bar{v}, \quad w = U_1\bar{w}, \quad p = \frac{\rho U^2}{\mathrm{Re}\,\varepsilon^2}\bar{p} \tag{2.2}$$

Here, l is the mean curvature of the solid surfaces, δ is the thickness of the layer, U_i and V_i are the longitudinal and transverse velocities of the lower ($i = 1$) and upper ($i = 2$) solid surfaces, respectively,

$\mathrm{Re} = U_1 l\rho/\mu$ is the Reynolds number, $\varepsilon = \delta/l$, and μ is the characteristic shear viscosity of the liquid which may correspond to one of the eight coefficients ($a_1, \ldots, a_8$) or may be a combination of them.

A dimensionless similarity number may be constructed

$$A = \delta\sqrt{\mu/\Theta} \tag{2.3}$$

which is analogous to the Reynolds number. Here, Θ is the characteristic momentum viscosity of the fluid. In obtaining the equations of motion for a lubricating layer it is assumed that

$$\varepsilon \ll 1, \quad \varepsilon \approx \frac{1}{\mathrm{Re}}, \quad \varepsilon \approx \frac{1}{l}\sqrt{\frac{\Theta}{\mu}} \tag{2.4}$$

If account is taken of (2.1), (2.3) and (2.4) in the material relations and the equations of motion, then, in the zeroth approximation, that is, when the terms $O(\varepsilon)$ are neglected, we can obtain

$$\begin{aligned}
&-\frac{\partial p}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \frac{\partial u}{\partial t}, \quad \frac{\partial p}{\partial y} = 0\\
&-\frac{\partial p}{\partial z} + \frac{\partial \sigma_{zy}}{\partial y} = \frac{\partial w}{\partial t}, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\\
&\frac{\partial \mu_{xy}}{\partial y} - \sigma_1 = 0, \quad \frac{\partial \mu_{yy}}{\partial y} - \sigma_2 = 0, \quad \frac{\partial \mu_{zy}}{\partial z} - \sigma_3 = 0\\
&\sigma_{xy} = \frac{1}{4}(b_1 + b_2^+ L_1^2 + b_2^- L_2^2 + 4a_3 L_1^2 L_2^2)\frac{\partial u}{\partial y} +\\
&+\frac{1}{4}(b_2^+ L_1 L_3 + 4a_3 L_1 L_2^2 L_3)\frac{\partial w}{\partial y} + \frac{1}{2} b_3 \dot{\Omega}_3 + \frac{a_6}{2}\left(L_1 \frac{\partial L_2}{\partial t} + L_2 \frac{\partial L_1}{\partial t}\right)\\
&\sigma_{zy} = \frac{1}{4}(b_2^+ L_1 L_3 + 4a_3 L_1 L_2^2 L_3)\frac{\partial u}{\partial y} + \frac{1}{4}(b_1 + b_2^- L_2^2 + b_2^+ L_3^2 + 4a_3 L_2^2 L_3^2)\frac{\partial w}{\partial y} -\\
&-\frac{1}{2} b_3 \dot{\Omega}_1 + \frac{a_6}{2}\left(L_3 \frac{\partial L_2}{\partial t} + L_2 \frac{\partial L_3}{\partial t}\right)\\
&\sigma_1 = -\frac{1}{2} b_3^+ L_1 L_3 \frac{\partial u}{\partial y} - \frac{1}{2}(2a_7 + b_3^- L_2^2 + b_3^+ L_3^2)\frac{\partial w}{\partial y} + b_3 \dot{\Omega}_1\\
&\sigma_2 = -\frac{b_3^-}{2} L_2\left(L_3 \frac{\partial u}{\partial y} + L_1 \frac{\partial w}{\partial y}\right) + b_3 \dot{\Omega}_2\\
&\sigma_3 = \frac{1}{2}(2a_7 + b_3^+ L_1^2 + b_3^- L_2^2)\frac{\partial u}{\partial y} + \frac{1}{2} b_3^- L_1 L_3 \frac{\partial w}{\partial y} + b_3 \dot{\Omega}_3\\
&b_1 = 2a_1 + 2a_7, \quad b_2^\pm = a_2 + a_8 \pm 2a_6, \quad b_3^\pm = a_8 \pm a_6, \quad b_3 = 2a_7 + a_8
\end{aligned} \tag{2.5}$$

In Eqs (2.5) we have again changed from dimensionless quantities to dimensional quantities and here the previous notation is retained for the pressure, the components of translational velocity and the force and momentum stress tensors. In the case of the intrinsic angular velocity vector in the thin-layer approximation, we have (to accuracy $O(\varepsilon)$)

$$\dot{\Omega}_i = L_n \frac{\partial L_m}{\partial t} \varepsilon_{inm} \tag{2.6}$$

3. BOUNDARY CONDITIONS

The boundary conditions have to be specified in order to solve the equations of motion of a liquid with momentum anisotropy. Using the "no-slip" hypothesis, the translational velocity field $\mathbf{v}$ on the solid surface s can be written as in conventional hydrodynamics

$$\mathbf{v}(\mathbf{r}, t)|_s = \mathbf{V}_s \tag{3.1}$$

where $\mathbf{V}_s$ is the translational velocity of the motion of the solid surface s. The boundary conditions for the vector $\mathbf{L}(\mathbf{r}, t)$ must reflect the mechanism of the interaction between the LMA and the solid surface. Since the details of this mechanism are far from clear, we shall start out in the first approximation from the assumption that the long axes of the molecules are "rigidly" orientated on the solid surface. In this case

$$\mathbf{L}(\mathbf{r}, t)|_s = \mathbf{L}_s \tag{3.2}$$

where $\mathbf{L}_s$ is the vector describing the orientation of the long axes of the molecules on the solid surface. Additionally, we assume that the initial translational velocity field and the initial orientation of the long axes of the molecules are specified, that is

$$\mathbf{v}(\mathbf{r}, t)|_{t=0} = \mathbf{V}_0(\mathbf{r}), \quad \mathbf{L}(\mathbf{r}, t)|_{t=0} = \mathbf{L}_0(\mathbf{r}) \tag{3.3}$$

Boundary conditions (3.1)–(3.3) enable one, in principle, to integrate the equations of motion of a LMA and to determine the translational velocity field $\mathbf{v}(\mathbf{r}, t)$ and the orientation of the long axes of the molecules $\mathbf{L}(\mathbf{r}, t)$.

Other assumptions are made in the literature concerning the theory of microstructural liquids with regard to the mechanism of the interaction between the liquid molecules and a solid surface, and other forms of boundary conditions have been formulated [9, 10].

4. A LAYER OF LMA BETWEEN PARALLEL PLATES

As an illustration of the rheological effects of a LMA we will consider its flow between parallel plates. To fix our ideas, we shall assume that the lower plate is fixed while the upper plate is spaced a distance h from the lower plate and moves along the x axis at a constant velocity U. The angle $\Phi = \Phi(y, t)$ is measured from the y axis. Allowing for the symmetry of the flow, we can then write

$$\mathbf{v} = u(y)\mathbf{e}_x, \quad \mathbf{L} = \mathbf{e}_x\sin\Phi + \mathbf{e}_y\cos\Phi \tag{4.1}$$

On substituting (4.1) into the equations of motion (2.5), we conclude that p = const. Without loss in generality it may be assumed that $p = 0$. We now introduce the scales for the dimensional quantities

$$[\eta] = \eta_\perp, \quad [u] = U, \quad [\theta] = \theta_\perp, \quad [y] = h, \quad [t] = h/U, \quad [\sigma] = U\eta_\perp/h$$
$$\eta_\perp = (b_1 + b_2^-)/4, \quad \eta_\parallel = (b_1 + b_2^+)/4 \tag{4.2}$$

The system comprising the equations of motion and the material relations and the boundary conditions and the initial conditions are then rewritten in the form

$$\frac{\partial}{\partial \bar{y}}\left(\bar{\eta}_1 \frac{\partial \bar{u}}{\partial \bar{y}} - \bar{\eta}_2 \frac{\partial \Phi}{\partial \bar{t}}\right) = 0 \tag{4.3}$$

$$\frac{\partial}{\partial \bar{y}}\left(\bar{\theta}\frac{\partial^2 \Phi}{\partial \bar{y}\partial \bar{t}}\right) + A^2\left(\bar{\eta}_2 \frac{\partial \bar{u}}{\partial \bar{y}} - \bar{\eta}_\omega \frac{\partial \Phi}{\partial \bar{t}}\right) = 0 \tag{4.4}$$

$$u|_{y=0} = 0, \quad u|_{y=1} = 1, \quad u|_{t=0} = u(y, 0) \tag{4.5}$$

$$\Phi|_{y=0} = \Phi_0, \quad \Phi|_{y=1} = \Phi_h, \quad \Phi|_{t=0} = \Phi(y, 0)$$

Here

$$\bar{\theta} = \bar{\theta}_\parallel \sin^2\Phi + \cos^2\Phi, \quad \bar{\eta}_1 = \cos^2\Phi + \bar{\eta}_\parallel \sin^2\Phi + \bar{\eta}_+ \sin^2\Phi\cos^2\Phi$$

$$\bar{\eta}_2 = \frac{1}{2}[(\bar{\eta}_\omega - \bar{\eta}_\parallel + 1)\cos^2\Phi + (\bar{\eta}_\omega + \bar{\eta}_\parallel - 1)\sin^2\Phi]$$

$$\bar{\eta}_\parallel = \frac{\eta_\parallel}{\eta_\perp}, \quad \bar{\eta}_+ = \frac{\eta_+}{\eta_\perp}, \quad \bar{\eta}_\omega = \frac{\eta_\omega}{\eta_\perp}, \quad \bar{\theta}_\parallel = \frac{\theta_\parallel}{\theta_\perp}, \quad A^2 = \frac{h^2\eta_\perp}{\theta_\perp}$$

$$\eta_+ = a_3, \quad \eta_\omega = b_3$$

Henceforth, the bar over dimensionless quantities is omitted. On integrating Eq. (4.3) with respect to y, we obtain

$$\eta_1 \frac{\partial u}{\partial y} - \eta_2 \frac{\partial \Phi}{\partial t} = \tau(t) \tag{4.6}$$

where τ depends on time if the long axes of the molecules "twist" with respect to the solid surface. In the case when they are rigidly clamped in accordance with condition (4.5), τ is independent of t.

Solving Eq. (4.6) for $\partial u/\partial y$ and substituting the result into (4.4), we obtain an equation for determining $\Phi(y, t)$

$$\frac{\partial}{\partial y}\left(\theta \frac{\partial^2 \Phi}{\partial y \partial t}\right) + A^2\left[\left(\frac{\eta_2^2}{\eta_1} - \eta_\omega\right)\frac{\partial \Phi}{\partial t} + \frac{\eta_2 \tau}{\eta_1}\right] = 0 \tag{4.7}$$

When

$$\eta_{\|} = 1, \quad \theta_{\|} = 1, \quad \eta_+ = 0$$

the solution of Eq. (4.7) can be obtained in analytic form

$$u = \frac{\eta_\omega}{2(2\alpha + \eta_\omega \Delta)}\left[\frac{4\alpha y}{\eta_\omega} - \sinh(\alpha y) + \Delta(1 - \cosh(\alpha y))\right]$$

$$\Phi = \Phi(y,0) + \frac{\alpha t}{2\alpha + \eta_\omega \Delta}[1 - \cosh(\alpha y) - \Delta \sinh(\alpha y)] \tag{4.8}$$

$$\Phi(0,0) = \Phi_0, \quad \Phi(h,0) = \Phi_h$$

$$\sigma_{xy}|_{y=0} = \tau = \frac{1 - \eta_\omega/4}{1 - [\eta_\omega \tanh(\alpha/2)]/(2\alpha)}$$

$$\alpha^2 = A^2 \eta_\omega \left(1 - \frac{\eta_\omega}{4}\right), \quad \Delta = \frac{1 - \cosh\alpha}{\sinh\alpha}.$$

Profiles of the intrinsic angular velocity of the liquid $\dot{\Omega} = \partial\Phi/\partial t$ as a function of the parameter A through a cross-section of the layer when $\eta_\omega = 0.5$ are shown in Fig. 1. It is seen that in thick layers ($A = 20$) the flow in the centre of the stream approximates to the classical case: the liquid particles rotate at a constant angular velocity, equal to the angular velocity of rotation, $\dot{\omega}$, of a part of the medium as a whole. The orientational action of the solid surfaces manifests itself solely in a narrow boundary layer in this case. Its thickness can be estimated from the law of the asymptotic approach of $\dot{\Omega}$ to $\dot{\omega}$. It follows from the second equality of (4.8) that

$$\dot{\Omega}_{max} \approx \tfrac{1}{2} - \exp(-\alpha/2)$$

If we limit ourselves to an accuracy of 0.1 and assume that $\alpha = 4$, we can obtain

$$h = 4\sqrt{\frac{\theta_\perp}{\eta_\perp \eta_\omega (1 - \eta_\omega/4)}}$$

that is, the thickness of the boundary layer of the liquid is determined by its rheological properties.

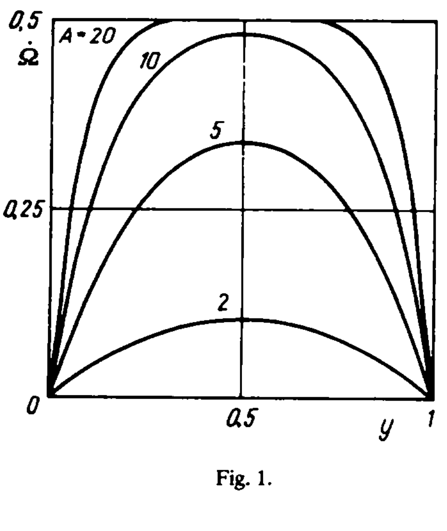

Fig. 1.

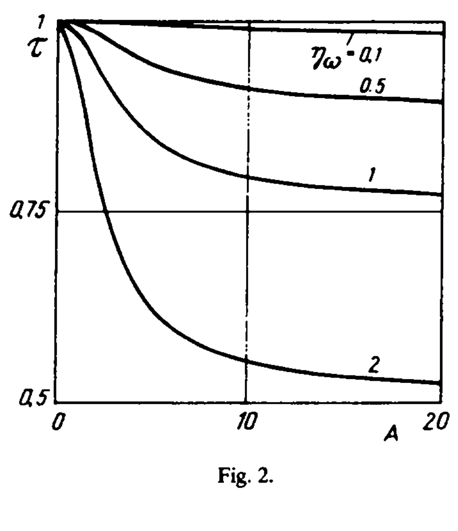

Fig. 2.

As the thickness of the layer decreases (as the parameter A becomes smaller), the orienting action of the solid surfaces propagates throughout the bulk of the layer; the boundary layers overlap and the angular velocities of rotation of the axes of anisotropy are "frozen", that is, $\dot{\Omega} \to 0$.

It is also seen from Fig. 1 that the angular velocity of rotation of the liquid particles is a maximum in the middle of the layer and decreases smoothly as one approaches the solid surfaces. In particular, it follows from this that the coefficient of birefringence of such a layer will decrease smoothly from the solid surface to the middle layer [11] which is in complete qualitative agreement with the available experimental data [1–4]. In the given range of parameters, the profile of the translational velocity u is only slightly different from a classical linear profile.

The shear stresses $\tau = \sigma_{xy}|_{y=0}$, acting on the solid surface at different values of the rotational viscosity are shown in Fig. 2. These graphs show that a layer of a LMA exhibits a dimensional effect: in wide gaps when $A \to \infty$, we have $\mu_e \to \eta_\perp(1 - \eta_\omega/4)$ and in narrow gaps, when $A \to 0$, the effective viscosity of the liquid $\mu_e \to \eta_\perp$. The dimensional effect on viscosity is greater the greater the rotational viscosity η_ω.

In the general case, that is, when account is taken of the anisotropy of the shear and momentum viscosities, both the intrinsic angular velocity as well as the translational velocity of the liquid turn out to be complex functions of time. An analysis of their effect on the basic rheological characteristics of a layer requires special consideration.

This research was carried out with financial support from the Russian Foundation for Basic Research (94-01-00450-a).

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Translated by E.L.S.